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# THE WELFARE CONSEQUENCES OF LEGAL SYSTEM IMPROVEMENT\*

BENJAMIN E. HERMALIN,<sup>†</sup> LARRY LI,<sup>‡</sup> AND TONY NAUGHTON<sup>§</sup>

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## ABSTRACT

This paper explores the consequences of an improving legal system on an economy that can also rely on relational (informal) contracting. We show such improvement can be welfare enhancing or welfare reducing. In the latter case, we show that often a mediocre legal system is worse than either a bad legal system or an excellent system, with the second often being superior to relational contracting. The paper identifies situations in which it pays to use both informal and formal contracting, with formal contracting serving to enhance relational contracting.

*Keywords:* formal and relational contracting and economic development.

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<sup>†</sup>University of California • Berkeley, CA, USA • email: hermalin@berkeley.edu.

<sup>‡</sup>School of Economics, Finance, & Marketing • RMIT University • Melbourne, VIC, Australia • email: larry.li@rmit.edu.au.

<sup>§</sup>School of Economics, Finance, & Marketing • RMIT University • Melbourne, VIC, Australia • email: tony.naughton@rmit.edu.au.

CONTENTS

1	INTRODUCTION	1
2	MODEL	4
3	FIRST BEST AND THE ONE-SHOT GAME	7
4	THE REPEATED GAME	9
5	CONCLUSION	15
	APPENDIX A: PROOFS	16
	REFERENCES	26

## 1 INTRODUCTION

All but the simplest economic activity require contracts. These contracts can be formal, relying on the capacity and reliability of the legal system, or relational in the sense that the strength of ongoing ties between the parties forms the basis of enforcement. Relational contracting can be observed to be particularly strong in economies where contract law and its enforcement are inadequate. Relational contracting, therefore, plays an important role in emerging or transition economies.<sup>1</sup> In developed economies, in contrast, most economic interactions of interest are governed by formal contracts.<sup>2</sup>

Institutional development is not like flipping a light switch; a country does not go overnight from a regime of weak legal institutions to one of strong legal institutions. Improvements in institutions is gradual. A naïve view might be that corresponding to this gradual improvement in the legal system would be an improvement in welfare. As, however, Schmidt and Schnitzer (1995) observed, there is a cost to improving the legal system—a better legal system can undermine relational contracting. The logic is straightforward: The better is formal contracting, the less dire the consequences of a relationship collapse, and, hence, the harder it is to maintain a relationship.<sup>3</sup> So if welfare under relational contracting is great enough, a modest improvement in formal contracting can yield equilibria with lower welfare.

On the other hand, the most developed economies appear to be those with strong legal systems and there is a large literature arguing that a well-functioning legal system is essential to economic prosperity (see, *e.g.*, Johnson et al., 2002, and Djankov et al., 2003). This suggests that a strong legal system is superior to relational contracting, or that a strong legal system is not as detrimental to relational contracting as Schmidt and Schnitzer might seem to suggest, or both.

In this paper, we develop a framework within which to model an improving legal system. Within this framework, we show, *inter alia*, that Schmidt and Schnitzer’s conclusions are arguably incomplete. Although a stronger legal system can undermine relational contracting, we show that conclusion could be

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<sup>1</sup>See, *e.g.*, Greif (1993), Kranton (1996), Kranton and Swamy (1999), and McMillan and Woodruff (1999a,b).

<sup>2</sup>See, *e.g.*, Hermalin et al. (2007). At the same time, Macaulay (1963) argued that informal ties between parties can strengthen formal contracting. Hence, even well-developed economies, such as Japan, can be observed to have extensive relationships at the core of business-to-business contracting—a point to which we return later.

<sup>3</sup>Specifically, Schmidt and Schnitzer assume formal contracting can achieve the same outcomes as informal contracting, but the use of formal contracts means incurring costs absent with informal contracting. So, in their model, “better formal contracting” means a reduction in those costs. In contrast, we restrict attention to situations in which outcomes supportable with informal contracts simply cannot be supported with formal contracts if the legal system is sufficiently poor, but the set of outcomes supportable by formal contracts expands as formal contracting becomes better (the legal system improves). As an abstraction, we assume the costs of the two kinds of contracting are the same.

As Bård Harstad observed in a discussion of an earlier draft of this paper, improving the legal system could *raise* the cost of its utilization (*e.g.*, expectations for how well contracts are drafted could rise and, with them, legal costs). We do not explore that possibility either.

dependent on assuming that relational and formal contracting is an either-or proposition. If we allow the parties to use both simultaneously, then we find that stronger formal contracting can, in some instances, strengthen relational contracting.<sup>4</sup> Specifically, by writing formal contracts that, in themselves, would not support desirable outcomes, the parties are able to ensure greater adherence to their relational contract.<sup>5</sup>

By allowing for the various effects of an improving legal system, our framework allows us to investigate questions about what path welfare takes as a legal system improves.<sup>6</sup> Even if the use of formal and informal contracts is *not* either-or, an improved legal system can still undermine relational contracting sufficiently to make welfare fall initially as the legal system improves.

Even if welfare dips as the legal system improves, we show that eventually, if the legal system becomes good enough, it will prove superior to relational contracting alone (unless the latter achieves the first best). Hence, the success of the developed world is consistent with the analysis we set forth below.

Our paper builds on the literature on relational contracting (see, *e.g.*, MacLeod, 2007, for a survey). Baker et al. consider a principal-agent relationship in which the parties observe both the agent's actual performance and a signal correlated with it. Critically, only the latter is verifiable. When the correlation between signal and actual performance is sufficiently good, it is no longer feasible for the parties to also employ a relational contract based on actual performance. Hence, as in Schmidt and Schnitzer, the ability to contract formally can eliminate informal contracting. If one interpreted stronger correlation as a more accurate court, then Baker et al. can be seen as reaching a result opposite to our result that, when both relational and formal contracts can be used, formal contracting can help support informal contracting. The difference arises because, in this interpretation of their model, what is at issue is how good the court is at observing what happened; in contrast, we assume the court can observe that breach occurred, but it may err in assigning blame, or reach a judgment after much delay, or is, in some other way, unable to guarantee the injured party receives its full compensation.<sup>7</sup> Given that a number of authors have suggested that it is not the ability of the court to see what happened, but

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<sup>4</sup>Baker et al. (1994) and Kvaløy and Olsen (2009) also investigate what happens if formal and informal contracting are used simultaneously. Their models differ from ours in a number of important ways, as discussed below.

<sup>5</sup>This reverses, in a sense, Macaulay's argument that relational contracting strengthens formal contracting.

<sup>6</sup>Kranton and Swamy (1999) show, in the case of British India, that improvements in the court system had pluses and minuses with respect to farm lending. Johnson et al. (2002), and Djankov et al. (2003), *inter alia*, consider empirically some of the issues associated with evolving legal systems. In particular, they document that different legal systems vary in quality and that more-developed economies tend to have higher quality legal systems. Johnson et al. find evidence that changes in the quality of the legal system have bearing on inter-firm interactions consistent with the theoretical predictions we set forth here.

<sup>7</sup>Djankov et al. note that the principal way in which courts are "low quality" is the delay with which they reach judgments.

rather its ability to enforce contracts, that should be at issue, our model could be a valuable addition to the literature.<sup>8</sup>

Attention should also be paid to Kvaløy and Olsen. Like us, they allow the parties to write formal contracts but rely on the ongoing relation to ensure compliance. In their model, the cost of formal contracting is endogenous. The more one party, the principal, expends on formal contracting, the better that contracting is in the sense that it will yield greater gross surplus in equilibrium. This should be contrasted with Schmidt and Schnitzer's model, in which the cost of formal contracting is independent of the quality of the formal contracts. An ongoing relationship permits spending less on the formal aspects of the contract than one would in a one-shot game. This threat of having to pay more should there be a breakdown in trust helps support the reputational relationship. Consistent with Schmidt and Schnitzer's finding, should the *marginal* cost of formal contracting shift down, then relational contracting is undermined. If one interprets the fall in marginal cost as arising from an improved legal system, then Kvaløy and Olsen's results capture one of the two effects we find: Like us (and Schmidt and Schnitzer), they find that the better formal contracting is as a substitute for informal contracting, the less effective informal contracting will be. But there is also an opposite effect: We show that, because better formal contracting also raises the immediate penalty for breach, better formal contracting can help to support relational contracting.

The remainder of the paper has the following organization. The next section introduces our model. Firms seek to cooperate with each other. Such cooperation is variable and will depend on the contracts (relational, formal, or both) in place. We model the legal system as an imperfect enforcer of contracts, where the quality of the legal system is proxied by the reliability and speed with which courts enforce contracts. Section 3 details what the first-best solution would be and what would happen if the parties interacted only once. The focus of Section 4 is on the situation in which the parties can maintain a relationship. Much of the analysis is spent analyzing how welfare is affected as the quality of the legal system improves. As noted above, the analysis depends, in part, on whether formal and relational contracting is an either-or proposition or both can be utilized simultaneously. In the latter case, there are circumstances in which welfare is strictly increasing in the quality of the legal system. In the former case, a non-monotonic relation between quality of the legal system and welfare will exist; initially, improvements in the legal system will reduce welfare, but ultimately welfare will begin to improve with the quality of the legal system. For sufficient improvement and assuming relational contracting does not achieve the first best, welfare will be greater than it could be under relational contracting. We conclude in Section 5. Proofs not given in the text are in Appendix A. Table A.1 lists the notation employed.

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<sup>8</sup>Hermalin (2008) observes that if the parties can renegotiate prior to a court case, then the court's accuracy in judging *what* happened could be largely immaterial. See also the surveys by Hermalin et al. (2007) and MacLeod for more on the broader issue of the relevance of the observable-verifiable distinction.

## 2 MODEL

### 2.1 BASIC ASSUMPTIONS

There are two parties (individuals, firms, etc.). Within each period, the timing is as follows:

1. An agreement on a contract (relational, formal, or both) for that period is reached.
2. Each party chooses the amount of cooperation,  $q \in \mathbb{R}_+$ , it will supply that period.
3. Payoffs are realized.

Let the payoff to party  $i$  be  $\beta(q_1, q_2) - c(q_i)$ , where  $\beta : \mathbb{R}_+^2 \rightarrow \mathbb{R}$  and  $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ . Assume the functions are twice continuously differentiable in all their arguments. We further assume:

- Cost and marginal cost are both increasing in  $q$ ; that is,  $c(\cdot)$  is increasing and strictly convex. To ensure interior maxima, we assume  $c'(0) = 0$ .
- The benefit of no cooperation is normalized to zero; that is,  $\beta(0, 0) = 0$ .
- The benefit function is symmetric; that is, for any  $q$  and  $q'$ ,  $\beta(q, q') = \beta(q', q)$ .
- At least on some margins, benefit increases in cooperation. Specifically,  $\partial\beta(q_i, q_j)/\partial q_i > 0$  whenever  $q_i \leq q_j$ ; at some points in the analysis, we assume the more stringent condition:

**Strong Increase:** For all  $q_i$  and  $q_j$ ,  $\partial\beta(q_i, q_j)/\partial q_i > 0$ .

- For a given sum of  $q_1$  and  $q_2$ , output is at least weakly greater if the  $q$ s are more equal rather than less equal. Formally, we assume that the function  $\beta$  is Schur concave.<sup>9</sup>

Among the functions that would satisfy these assumptions are those in the class:

$$\beta(q_1, q_2) = f(q_1) + f(q_2),$$

where  $f(\cdot)$  is concave. Another example is

$$\beta(q_1, q_2) = (q_1 + q_2)^{3/2} + \exp(-(q_1 - q_2)^2).$$

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<sup>9</sup>Schur concavity means that if  $(q_1, q_2)$  majorizes  $(q'_1, q'_2)$ , then  $\beta(q_1, q_2) \leq \beta(q'_1, q'_2)$ . In our context, majorization can be defined as  $(q_1, q_2)$  majorizes  $(q'_1, q'_2)$  if  $|q_1 - q_2| > |q'_1 - q'_2|$  and  $q_1 + q_2 = q'_1 + q'_2$ . More generally, majorization is defined as follows. Consider two vectors of dimension  $N$ ,  $\mathbf{x}$  and  $\mathbf{y}$ . The vector  $\mathbf{x}$  majorizes  $\mathbf{y}$  if, when the elements of each vector are ordered largest to smallest,  $\sum_{n=1}^k x_n \geq \sum_{n=1}^k y_n$  for all  $k < N$  and  $\sum_{n=1}^N x_n = \sum_{n=1}^N y_n$ . Note the similarity between the concepts of majorization and second-order stochastic dominance.

Note that, although the latter example is a Schur-concave function, it is not a concave function.

Although we recognize real-life relations could be asymmetric, assuming  $\beta$  is a symmetric function greatly simplifies the analysis insofar as we don't continually need to develop parallel conditions for two kinds of parties. A further advantage is it allows us to further assume  $\beta$  is Schur concave (a necessary condition for Schur concavity is symmetry). Schur concavity, in turn, essentially serves to guarantee symmetric equilibria, which again avoids the complications of having to develop parallel conditions and possible multiplicity of equilibria. It also seems a reasonable assumption in and of itself because it strikes us that in many relationships getting the partner supplying less effort to increase his or her effort improves the outcome by more than would an equal increase in effort by the partner supplying more effort (this is essentially what Schur concavity entails).

We also assume there exists a positive and finite value  $q_M$  such that

$$2 \frac{\partial \beta(q, q)}{\partial q} - c'(q) < 0 \quad (1)$$

for all  $q > q_M$ . As will be seen, this ensures that the parties always wish to choose a finite level of cooperation.

We assume that the two parties are the “only game in town” insofar as neither has the opportunity to trade with a third party. This assumption means that a breakdown in relational contracting is *not* punished by terminating the relationship, but by use of formal contracts. We recognize that, in certain situations, parties could terminate their relationship and search for new partners. If, however, it is relatively easy to find new partners, then it will be difficult to maintain relational contracts in the first place because the outside option is presumably as good or close to as good as the current relationship. (This assumes reputations cannot be carried by word of mouth.) Consequently, relational contracting could be most prevalent when parties tend to find each other the only game in town. Alternatively, if reputation could be carried by word of mouth, so a player that breached its relational contract could be blackballed by its former partner; or if a newly partnerless player is mistrusted, then such a player could find itself limited to formal contracts with new partners only. In this interpretation, the continuation payoffs considered below that a breaching party gets from formal contracting are simply the payoffs from contracts it signs with a new partner (or new partners). In this way, the analysis below can be extended to situations in which parties have potentially multiple partners.

## 2.2 THE LEGAL SYSTEM

The parties may wish to enter into a formal contract that specifies the levels of  $q_1$  and  $q_2$ . Consider a specific contract  $\langle \bar{q}_1, \bar{q}_2 \rangle$ . In what follows, there is no loss of generality in considering party 1 to be the contract breacher and party 2

to be the injured party. Imagine that party 1 breaches by choosing  $q_1 < \bar{q}_1$ .<sup>10</sup> We assume that the court is imperfect. Specifically, in the event of contract breach it awards damages to the non-breaching (injured) party (here, party 2) with probability  $\theta \in [0, 1)$ . Note  $\theta = 0$  could be viewed as equivalent to no court system. The parameter  $\theta$  is common knowledge at the time the parties contract. The parameter  $\theta$  can be thought of as a measure of the quality of the legal system. We discuss its interpretation at greater length at the end of this subsection.

We assume, in keeping with the law's abhorrence of penalties in private contracts (see *e.g.*, Hermalin et al., 2007, §5.3), that there are limits on the amount of damages  $D$ . In particular, and consistent with practice in many courts, we limit damages so as not to exceed the actual loss suffered. In legal terms, we are focusing on expectation damages; that is,

$$D \leq \beta(\bar{q}_1, \bar{q}_2) - \beta(q_1, \bar{q}_2).$$

Damage claims greater than this will be dismissed as punitive by the courts (consistent with common practice). Beyond its realism, limiting attention to expectation damages facilitates the analysis. That said, we note that our basic conclusions would still apply if there were no court-imposed limits on penalties, but parties can go bankrupt. For low enough court quality, it would still be the case that relational contracting would dominate legal contracting, which is what is essential for our analysis.

In equilibrium, the parties will honor the contract. Hence, on the equilibrium path, penalties aren't paid. It is, therefore, without loss of generality to assume the parties set the maximum penalty in their contract so as to achieve the maximum deterrence effect. Hence, they utilize expectation damages. The expected payoffs to the breaching party (here, 1) and the payoff to the injured party (here, 2) are, respectively,

$$\begin{aligned} \Pi_1 &= (\theta + 1)\beta(q_1, \bar{q}_2) - \theta\beta(\bar{q}_1, \bar{q}_2) - c(q_1) \quad \text{and} \\ \Pi_2 &= (1 - \theta)\beta(q_1, \bar{q}_2) + \theta\beta(\bar{q}_1, \bar{q}_2) - c(\bar{q}_2). \end{aligned} \quad (2)$$

Our model of the legal system has a number of interpretations. One is that the court always reaches the correct verdict, but there is uncertainty over whether the court will hear the case or be able to enforce its judgment or both. An alternative, but related, interpretation is that the court accurately assigns guilt, but is subject to delays, so the plaintiff only recovers damages with a lag (*i.e.*, in this interpretation  $\theta$  is the relevant discount factor). A third interpretation is that only the plaintiff can receive damages (similar to the system in the United States), but there is uncertainty as to whether the court will arrive at the correct verdict. A final interpretation, although one

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<sup>10</sup>A "breach" in which party 1 chooses  $q_1 > \bar{q}_1$  is of no interest given that party 2 would not be harmed by such a deviation and would, thus, have no grounds on which to sue party 1. Moreover, in equilibrium, the parties would never agree to a contract in which party 1 had incentives to cooperate more than called for by the contract.

that requires a change in variables, is that the court always arrives at a verdict, but there is uncertainty as to whether the court will arrive at the correct verdict. Whichever party it judges to be responsible for breach must pay the other party damages (hence, sometimes, the plaintiff must pay the defendant, a possibility allowed for in some legal systems). If one adopts this last interpretation and  $\xi$  is the probability that the court rules for the injured party (party 2), then identical conclusions to those obtained below will be reached by making the transformation  $\xi = \frac{1}{2}(1 + \theta)$  (note, this restricts,  $\xi \in [1/2, 1)$ ; the injured party is at least as likely to receive damages as the breaching party).<sup>11</sup>

### 3 FIRST BEST AND THE ONE-SHOT GAME

We begin our analysis by determining what the first-best outcome would be and what would, instead, result were the interaction between the parties single shot. To that end, the following lemma will prove useful.

**Lemma 1.** *Under the above assumptions about the benefit function,  $\beta : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ , the following are true with regard to the expression*

$$\zeta\beta(q_1, q_2) - c(q_1) - c(q_2), \quad (3)$$

$\zeta \in (0, 2]$ :

- (i) *If  $(q'_1, q'_2)$  has the same sum as  $(q''_1, q''_2)$ , but is more disperse (i.e.,  $(q'_1, q'_2)$  majorizes  $(q''_1, q''_2)$ —see footnote 9 supra), then (3) is greater given  $(q''_1, q''_2)$  than given  $(q'_1, q'_2)$ .*
- (ii) *There exists a finite level of cooperation,  $q^*(\zeta)$ , such that (3) is maximized if each party chooses  $q^*(\zeta)$ .*
- (iii)  *$\zeta > \zeta'$  implies  $q^*(\zeta) > q^*(\zeta')$ .*
- (iv) *Holding one party's level of cooperation fixed, there exists a finite level of cooperation for the other party that maximizes (3).*
- (v) *That level of cooperation in part (iv) is increasing in  $\zeta$ .*

Joint payoffs are

$$\beta(q_1, q_2) - c(q_1) + \beta(q_1, q_2) - c(q_2) = 2\beta(q_1, q_2) - c(q_1) - c(q_2). \quad (4)$$

From the lemma, there exists a finite  $q^*(2)$  that, if each party chooses it, maximizes joint payoffs.

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<sup>11</sup>Another way of modeling legal-system quality, not pursued here, would be in terms of how corrupt the legal system is. In an article that looks at consumers' incentives to invest in learning about firms' reputations prior to purchase to avoid low-quality goods versus relying on being able to sue a firm that cheats them, Dhillon and Rigolini (2011) endogenize, to an extent, court quality by looking at the incentives of firms to bribe the court when they are sued by consumers for selling low quality goods. In that article, court quality varies in equilibrium with how costly it is for firms to provide high quality.

An individual party's payoff is, however,

$$\beta(q_1, q_2) - c(q_i), \quad (5)$$

$i = 1$  or  $2$ . By definition, it must be that  $q^*(2)$  is the solution to

$$\max_q 2\beta(q, q^*(2)) - c(q).$$

From parts (iv) and (v) of the lemma, there exists a level of cooperation less than  $q^*(2)$  that maximizes party  $i$ 's individual profit when the other party chooses  $q^*(2)$  (it is as if  $\zeta$  went from 2 to 1). In other words, maximizing their individual payoffs, the parties will not choose, in equilibrium, the level of cooperation that maximizes their joint payoffs. This establishes:

**Proposition 1.** *Were the parties to interact once, they would not choose a level of cooperation that maximized their joint payoffs.*

Proposition 1 is the familiar result that team members (here, the parties) underinvest relative to the first best because each team member bears 100% of the cost of his investment, but reaps only a fraction of the total benefit his investment produces (see, *e.g.*, Holmstrom, 1982). This problem is sometimes referred to as free-riding; ultimately it is a reflection of the well-known result that parties expend too little effort, from a welfare perspective, on activities that generate positive externalities.

For future reference, define

$$q^{\text{BR}}(q, \zeta) = \operatorname{argmax}_x \zeta\beta(x, q) - c(x). \quad (6)$$

Observe  $q^{\text{BR}}(q, 1)$  is a party's best response to the other party's choosing  $q$  assuming that he wishes to maximize a single period's profits only (assuming no contract).

**Lemma 2.**  $q^{\text{BR}}(q^*(\zeta), \zeta) = q^*(\zeta)$ .

Lemma 2 implies that a Nash equilibrium of the game played once, absent any contract, is for both parties to choose  $q^*(1)$ .

Consider the one-shot game with contracts, as outlined in Section 2.2. For party  $i$  to honor a contract calling for the parties to play  $\langle \bar{q}_i, \bar{q}_j \rangle$  it must be that

$$\beta(\bar{q}_i, \bar{q}_j) - c(\bar{q}_i) \geq \max_q (\theta + 1)\beta(q, \bar{q}_j) - \theta\beta(\bar{q}_i, \bar{q}_j) - c(q), \quad (7)$$

where the right-hand side follows from (2). We can rearrange (7) to reexpress the condition as

$$(\theta + 1)\beta(\bar{q}_i, \bar{q}_j) - c(\bar{q}_i) \geq \max_q (\theta + 1)\beta(q, \bar{q}_j) - c(q). \quad (8)$$

The following is immediate:

**Lemma 3.** *A contract  $(\bar{q}_1, \bar{q}_2)$  will be honored in equilibrium only if  $\bar{q}_i = q^{BR}(\bar{q}_j, \theta + 1)$  and  $\bar{q}_j = q^{BR}(\bar{q}_i, \theta + 1)$ .*

Although it is not essential for the subsequent analysis, it facilitates that analysis if we restrict attention to situations in which  $q^*(\zeta)$  is unique for all  $\zeta \in [1, 2]$ . To this end, we assume

**Assumption 1.** *The univariate function  $\mathbb{R}_+ \rightarrow \mathbb{R}$  defined by  $q \mapsto \zeta\beta(q, q) - 2c(q)$  is a strictly concave function (of  $q$ ) for all  $\zeta \in [1, 2]$ .*

We can now establish:

**Proposition 2.** *If the quality of the legal system is  $\theta$ ,  $\theta \in [0, 1)$ , then the only formal contract that will be honored as a pure-strategy equilibrium is the one that calls on each party to choose level of cooperation  $q^*(\theta + 1)$ .*

Proposition 2 has a few implications. First, given Lemma 1(v), it implies that the equilibrium level of cooperation in a one-shot game is greater the the better is the legal system (*i.e.*, the greater is  $\theta$ ). Second, if the court has minimum quality or non-existent (*i.e.*,  $\theta = 0$ ), then the outcome is identical to the outcome were no contract in force. Third, if the court is perfect,  $\theta = 1$ , then the outcome given a contract is the first-best outcome. Finally, given Assumption 1, the function  $\beta(q, q) - c(q)$  is strictly concave, so the closer  $q^*(\theta + 1)$  is to  $q^*(2)$ , the greater is welfare; in other words, the better the legal system (*i.e.*, the greater is  $\theta$ ), the greater is welfare.

## 4 THE REPEATED GAME

We now consider an infinitely repeated game. Assume the parties have the same discount factor,  $\delta$ .

A critical issue is whether we view relational and formal contracts as an either-or proposition or whether we allow for both to be used simultaneously. That is, can the parties have a relational and formal contract in force simultaneously or, alternatively, can they have only one or the other in force?

It might seem that the either-or case is the less empirically relevant case. This could well be true, but one can imagine an extension of our model in which there are fixed costs of writing formal contracts or using the courts. Hence, it is worth knowing how well the parties could do relying solely on relational contracts. Moreover, doing so permits us to highlight how allowing the parties to use both yields results that differ from earlier analyses that took an either-or approach.

Suppose, first, that choice of contract type is an either-or proposition. So, if the parties are operating under a relational contract, there can be no formal contract. The parties can, however, revert to using formal contracts should their relational contract collapse. In a world of either-or contracting, a level of

cooperation  $\hat{q}$  can be sustained in equilibrium if

$$\begin{aligned} (\beta(\hat{q}, \hat{q}) - c(\hat{q})) \frac{1}{1-\delta} &\geq \left( \max_q \beta(q, \hat{q}) - c(q) \right) \\ &+ \frac{\delta}{1-\delta} \underbrace{\left( \beta(q^*(\theta+1), q^*(\theta+1)) - c(q^*(\theta+1)) \right)}_{\pi_{\text{CON}}(\theta)}; \end{aligned}$$

or, rearranging, if

$$\beta(\hat{q}, \hat{q}) - c(\hat{q}) \geq (1-\delta) \left( \max_q \beta(q, \hat{q}) - c(q) \right) + \delta \pi_{\text{CON}}(\theta). \quad (9)$$

Ideally, the parties wish to sustain the first-best level of cooperation,  $q^*(2)$ . Because there is a finite maximum to the optimization program embedded in (9), it follows, by taking the limit of (9) as  $\delta \rightarrow 1$ , that there exists, for each  $\theta < 1$ , some discount factor such that the first best is supportable via relational contracts.

The following result follows readily from (9):

**Proposition 3.** *Assume the use of formal and relational contracts is either-or. For any quality of the legal system,  $\theta \in [0, 1)$ , there exists a cutoff,  $\delta^*(\theta) < 1$ , such that first best is supportable for all discount factors,  $\delta$ , such that  $\delta \geq \delta^*(\theta)$  and unsupportable for all  $\delta < \delta^*(\theta)$ . The function  $\delta^*(\cdot)$  is increasing and  $\lim_{\theta \rightarrow 1} \delta^*(\theta) = 1$ .*

In words, the better is the legal system, the higher the discount factor must be to sustain the first-best level of cooperation under relational contracting.

What if the discount factor is below the cutoff necessary to sustain the first best (*i.e.*, what if  $\delta < \delta^*(\theta)$ )? It can still be possible to support some cooperation greater than  $q^*(\theta+1)$  (the fallback level). To see this, suppose that  $\delta$  is just a bit less than  $\delta^*(\theta)$ . Suppose  $\hat{q}$  in (9) is reduced slightly from  $q^*(2)$ . By the envelope theorem and assuming the strong increase condition,

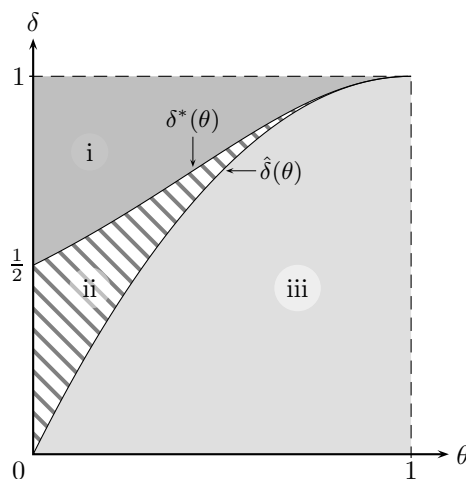
$$\frac{\partial}{\partial \hat{q}} \left( \max_q \beta(q, \hat{q}) - c(q) \right) = \frac{\partial \beta(q, \hat{q})}{\partial \hat{q}} > 0.$$

Because  $q^*(2)$  maximizes  $2\beta(q, q) - 2c(q)$  and, thus,  $\beta(q, q) - c(q)$ , we have

$$\frac{d}{d\hat{q}} (\beta(\hat{q}, \hat{q}) - c(\hat{q})) \approx 0$$

for  $\hat{q}$  near  $q^*(2)$ . It thus follows that reducing  $\hat{q}$  from  $q^*(2)$  leaves the left-hand side of (9) basically unchanged, but lowers the right-hand side of (9); hence, if (9) just failed to hold at  $\hat{q} = q^*(2)$ , it should hold for some  $\hat{q} < q^*(2)$  but “close.” Formally, we have

**Lemma 4.** *Assume the use of formal and relational contracts is either-or. Assume too that the “Strong Increase” condition holds. For any quality of the legal*



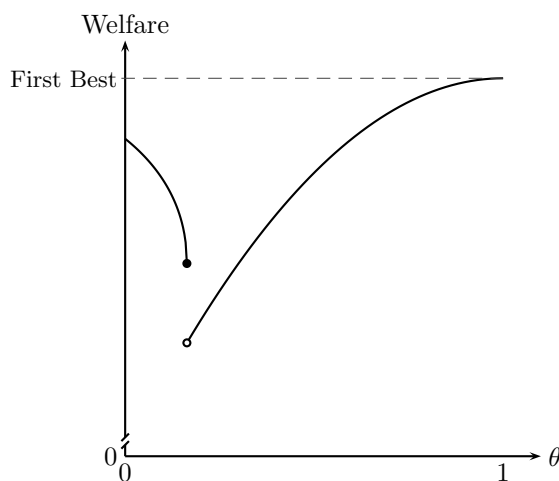
**Figure 1:** Equilibria by Region: (i) first best can be achieved; (ii) first best cannot be achieved, but relational superior to formal contracting; and (iii) formal contracting only. Figure drawn assuming  $\beta(q_1, q_2) = q_1 + q_2$  and  $c(q) = q^2/2$ .

system,  $\theta \in [0, 1)$ , there exists a cutoff,  $\hat{\delta}(\theta) < 1$ , such that a level of cooperation greater than that supportable by a formal contract is supportable via a relational contract for all  $\delta \geq \hat{\delta}(\theta)$ ; however, no level of cooperation greater than that supportable by a formal contract is supportable via a relational contract if  $\delta < \hat{\delta}(\theta)$ . The function  $\hat{\delta}(\cdot)$  is increasing and  $\lim_{\theta \rightarrow 1} \hat{\delta}(\theta) = 1$ .

Proposition 3 and Lemma 4 imply that  $\theta$ - $\delta$  space can be divided into three regions (see Figure 1): (i) a region in which the first best can be achieved; (ii) a region in which the first best cannot be achieved, but relational contracting is superior to formal contracting; and (iii) a region in which the equilibrium is characterized by formal contracting alone.

The existence of region (ii) offers one explanation for how, when the legal system is weak, relational contracting is superior to formal contracting, but how a vastly improved legal system can outperform relational contracting. Specifically, with regard to Figure 1, suppose  $\delta \in (0, 1/2)$ . When the legal system is at a minimum (*i.e.*,  $\theta = 0$ ), relational contracting is better than formal contracting. However, the welfare associated with relational contracting is less than the first-best level and will be so regardless of the quality of the legal system. When the legal system, however, becomes high quality ( $\theta \rightarrow 1$ ), then welfare under formal contracting approaches the first best.

Figure 2 plots the equilibrium level of welfare as a function of the quality of the legal system under the same functional assumptions as Figure 1 with  $\delta$  set equal to  $3/10$ . For low  $\theta$  (to the left of the discontinuity), a relational contract is used in equilibrium. For  $\theta$  high enough (to the right of the discontinuity),



**Figure 2:** Equilibrium Welfare as a Function of the Quality of Legal System. Same functional assumptions as Figure 1.  $\delta = 3/10$ . Scales of horizontal and vertical axes are not the same.

a formal contract is used in equilibrium. In turn, the relation between welfare and quality of the legal system is non-monotonic: Improving a low-quality legal system can reduce welfare, while improving a moderate-quality legal system can raise welfare.<sup>12</sup>

What if the use of relational and formal contracting is not either-or? In particular, even if a formal contract *alone* cannot induce the desired level of cooperation, it can serve to reinforce the relational contract by reducing the benefit of renegeing on the promised level of cooperation by imposing damage payments. Using (2), the condition for the parties to cooperate at a level  $\hat{q}$  given that failure to do so results in both the breaching party having to pay, with probability  $\theta$ , expectations damages *and* the end of relational contracting

<sup>12</sup>Note that Figure 2 (and Figure 3 *infra*) are comparative static results. In particular, they are thus different than an analysis that assumed the parties were aware the legal system was improving. Were that the case, then, being forward looking, anticipated future improvements and their consequent effects on relational contracting would be taken into account in the present, which could change the relational contracts agreed to in the present. That noted, if—as seems reasonable—the parties couldn't anticipate when improvements would occur and if the average rate of improvement were sufficiently slow, then the effect on the relational contracts signed would be fairly minimal and these figures would fairly accurately reflect the time path of welfare even with forward-thinking parties.

is

$$\begin{aligned} (\beta(\hat{q}, \hat{q}) - c(\hat{q})) \frac{1}{1 - \delta} \geq & \left( \max_q (\theta + 1) \beta(q, \hat{q}) - \theta \beta(\hat{q}, \hat{q}) - c(q) \right) \\ & + \frac{\delta}{1 - \delta} \left( \beta(q^*(\theta + 1), q^*(\theta + 1)) - c(q^*(\theta + 1)) \right); \end{aligned}$$

or, rearranging, the condition for cooperation can be written as

$$\beta(\hat{q}, \hat{q}) - c(\hat{q}) \geq (1 - \delta) \left( \max_q (\theta + 1) \beta(q, \hat{q}) - \theta \beta(\hat{q}, \hat{q}) - c(q) \right) + \delta \pi_{\text{CON}}(\theta). \quad (10)$$

Expression (10) is similar to (9) and identical if  $\theta = 0$  (i.e., for the lowest quality legal system). Consistent with the intuition given earlier, we can show the following.

**Lemma 5.** *Suppose a level of cooperation  $\hat{q}$ ,  $q^*(\theta + 1) < \hat{q} \leq q^*(2)$ , can be supported by a relational contract for a given discount factor and quality of the legal system,  $\theta$ , when the use of relational contracts and formal contracts is either-or. Then that level of cooperation can be supported by a relational contract “backed up” by a formal contract for the same discount factor and the same quality of the legal system.*

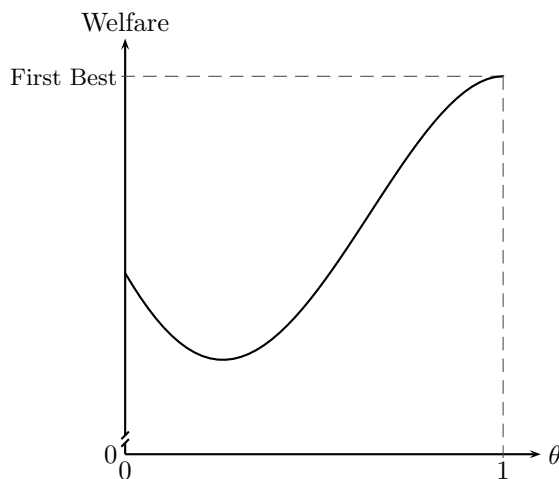
A corresponding result to Proposition 3 when both relational and formal contracting can be employed is

**Lemma 6.** *Assume that formal and relational contracts can be used simultaneously (i.e., the former “backs up” the latter). For any quality of the legal system,  $\theta \in [0, 1)$ , there exists a cutoff,  $\delta^{**}(\theta) < 1$ , such that the first best is supportable for all  $\delta \geq \delta^{**}(\theta)$  and unsupportable for all  $\delta < \delta^{**}(\theta)$ . Furthermore,  $\delta^{**}(0) = \delta^*(0)$  and  $\delta^{**}(\theta) < \delta^*(\theta)$  for all  $\theta \in (0, 1)$ .*

The astute reader will note an important difference between Proposition 3 and Lemma 6: We make no claim that  $\delta^{**}(\cdot)$  is increasing. In fact, it need not be, as summarized by the following result:

**Proposition 4.** *Suppose that formal contracts can be used to back up relational contracts. An improvement in the legal system can reduce, leave unchanged, or increase the space of parameters such that full efficiency can be achieved. As examples, suppose the cost of cooperation function is  $c(q) = q^2/2$ . The discount-factor cutoff,  $\delta^{**}(\theta)$ , for which full efficiency can be supported is*

- (i) increasing in the quality of the legal system if  $\beta(q_1, q_2) = \log(q_1 + q_2 + 1)$ ;
- (ii) constant with respect to the legal system if  $\beta(q_1, q_2) = q_1 + q_2$ ; and
- (iii) decreasing in the quality of the legal system if  $\beta(q_1, q_2) = q_1 + q_2 - (q_1 - q_2)^2$ .



**Figure 3:** Welfare can still be non-monotonic in the quality of the legal system even when formal contracts back up relational contracts. Figure drawn assuming  $\beta(q_1, q_2) = q_1 + q_2 - (q_1 - q_2)^2$ ,  $c(q) = q^2/2$ , and  $\delta = 3/10$ . Scales of horizontal and vertical axes are not the same.

Parts (ii) and (iii) of Proposition 4 shows that the prediction of Figure 2, namely that improvements in the quality of the legal system reduce welfare, at least in a neighborhood of minimum quality, could be an artifice of the assumption, underlying that figure, that the use of relational contracting and formal contracting is either-or. If formal contracts can be used to back up relational contracting, then welfare can be unaffected by improvements in the legal system (case ii) or benefitted directly (case iii). On the other hand, there is no guarantee that welfare is not reduced by an improvement of the legal system even outside the either-or context.

From Lemma 4, when formal and relational contracting is either-or, there exists a discount-factor cutoff,  $\hat{\delta}(\theta)$ , such that relational contracting is employed if and only if  $\delta \geq \hat{\delta}(\theta)$ ; recall Figure 1. When, instead, formal contracts can be used to back up relational contracts, relational contracting will be employed in equilibrium for all positive discount factors:

**Proposition 5.** *Suppose that formal contracts can be used to back up relational contracts. In equilibrium, the parties will always employ a relational contract backed-up by a formal contract rather than just a formal contract provided the discount factor is positive.*

Given, from Proposition 5, that formal contracts will always be used to back up relational contracts, it follows that improving the legal system has two, conflicting, effects on welfare: (i) it makes formal contracts a better back up, thus increasing welfare; but (ii), as in the either-or case, it also makes the

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alternative to relational contracting better, which reduces welfare. Hence, a non-monotonic relation between the quality of the legal system and welfare can still exist—see Figure 3. The one consequence, though, of using formal contracts as backups is that there is no longer a discontinuity in welfare as a function of the quality of the legal system (*i.e.*, compare Figures 2 and 3).

## 5 CONCLUSION

In this paper, we have presented a model in which interacting parties' choices between relational cooperation and formal contracting, or the decision to use both, is influenced by the quality of the legal system.

When the legal system is poor, parties need to rely on relational contracting to facilitate cooperation. Similar to the findings of Baker et al. (1994) and Schmidt and Schnitzer (1995), a better legal system can erode the effectiveness of relational contracting when the use of relational contracting or formal contracts is an either-or proposition. That is, improving the legal system can be welfare reducing. The either-or assumption could, however, be suspect in some settings. Hence, this paper explores what happens when formal contracts can serve to backup relational contracts—the parties always write formal contracts but adherence to them is essentially a function of repeated play, not the threat of legal sanctions. The same erosion seen in the either-or case may still occur, but whether it does depends on the details of how cooperation affects the parties' payoffs. Also, unlike earlier literature, this paper shows that when relational contracting cannot achieve the first best, there comes a point when improving the legal system is welfare enhancing.

In terms of policy implications, this paper suggests that a prescription of a better legal system need not always lead to better economic performance, at least in the short run. The analysis further indicates that a quantum improvement in the legal system could be preferable to incremental improvements. Of course, speeding the improvement in the legal system would naturally seem a good thing, but beyond the obvious benefit of speeding up when the returns of an improved legal system are enjoyed, a fast change reduces the period of time over which economic performance actually decreases. Moreover, to the extent reduced economic performance could undermine support for legal reforms or limit the funding for such reforms, a quantum leap may be necessary from a political-economy perspective. Future work may wish to explore in greater detail such political-economy issues; in particular, could incremental change “stall,” leading to long-term economic under performance?

Empirically, the paper suggests that measures of legal system effectiveness could have a U-shaped relation to economic performance. It also suggests that economies with strong traditions of personal ties among parties or strong personal-ties networks could outperform those without, at least in early stages of development. Of course, we recognize that the real world is far messier than any model, and hence any empirical analysis must take into account many important factors left out of our model. Hence, we view our paper as shedding light on some aspects of a complex reality, not as the definitive explanation for

all of it.

In terms of the modeling itself, we recognize more could be done. For instance, in our analysis, we limited attention to supporting symmetric levels of cooperation (*i.e.*,  $q_1 = q_2$ ) between the parties.

In our comparative statics analysis, we have effectively treated the players as naïve about there being an improving legal system (see footnote 12 *supra*). The modeling benefit of assuming such naïveté is that the repeated games can be treated as stationary, which greatly facilitates their analysis. Assuming less naïve players could lead to a more interesting dynamic analysis, although we note that, as long as the players cannot anticipate the timing of legal-system improvements, substantively similar conclusions would be reached. If the timing were anticipated, then—at least when the use of formal and relational contracts is an either-or proposition—an announcement of future reform could cause the immediate collapse of relational contracting. In this case, the analysis would be similar to that illustrated by Figure 2 except that the drop in welfare would occur immediately and welfare would be a function solely of the quality of the legal system.

In our modeling, we treat the legal system as somewhat of a black box; an improved legal system is simply taken to be one that is more likely to reach and enforce the correct decision.<sup>13</sup> How a legal system improves in this manner is not formally modeled. One possibility is that access to the legal system is rationed, so that the parameter  $\theta$  has to do with the probability of a case being heard. Rationing could also occur through delays; that is, all cases are heard, but many with great delay. In this case, as noted earlier, the parameter  $\theta$  could be interpreted as a discount factor on damage payments. Another way in which a legal system could improve is by becoming less corrupt. Although one could twist the model here to accommodate that form of improvement (*e.g.*, because of corruption, the contract breacher can reach a settlement with the injured party for less than the full damages, with the reduction being greater the more corrupt the legal system), it is more natural to see corruption as a cost of contracting—somewhat along the lines of Schmidt and Schnitzer (1995). This would change the modeling in some ways, but not, we believe, the substantive conclusions reached.

We are confident, therefore, that, while the modeling extensions and complications discussed in this section are important and will have implications for the details of the model and its predictions, the broader insights would be robust to these issues.

## APPENDIX A: PROOFS AND ADDITIONAL MATERIAL

Some of the analysis in this paper relies on the following well-known revealed-preference result, which is worth stating once, at a general level, for the sake of

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<sup>13</sup>A further issue is that, even restricting attention to formal contracting only, a more accurate legal system need not always be welfare improving; see Hermalin (2008). We abstract from the issues raised there.

completeness and to avoid unnecessary repetition. In what follows, the convention of using subscripts to denote partial derivatives is employed.

**Lemma A.1.** *Let  $f(\cdot, \cdot) : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a twice differentiable function. Suppose that  $f_{12}(\cdot, \cdot)$  has a constant sign. Let  $\hat{x}$  maximize  $f(x, z)$  and let  $\hat{x}'$  maximize  $f(x, z')$ , where  $z > z'$ . Then  $\hat{x} \geq \hat{x}'$  if  $f_{12}(\cdot, \cdot) > 0$  and  $\hat{x} \leq \hat{x}'$  if  $f_{12}(\cdot, \cdot) < 0$ . The inequalities are strict if either  $\hat{x}$  or  $\hat{x}'$  (or both) are interior maxima.*

**Proof:** By the definition of an optimum (revealed preference):

$$f(\hat{x}, z) \geq f(\hat{x}', z) \text{ and} \quad (11)$$

$$f(\hat{x}', z') \geq f(\hat{x}, z'). \quad (12)$$

Expressions (11) and (12) imply

$$\begin{aligned} 0 &\leq (f(\hat{x}, z) - f(\hat{x}', z)) - (f(\hat{x}, z') - f(\hat{x}', z')) \\ &= \int_{\hat{x}'}^{\hat{x}} (f_1(x, z) - f_1(x, z')) dx = \int_{\hat{x}'}^{\hat{x}} \left( \int_{z'}^z f_{12}(x, y) dy \right) dx, \end{aligned}$$

where the integrals follow from the fundamental theorem of calculus. Because the direction of integration is left to right, the inner integral in the rightmost term is positive if  $f_{12}(\cdot, \cdot) > 0$  and negative if  $f_{12}(\cdot, \cdot) < 0$ . It follows that the direction of integration in the outer integral must be weakly left to right (*i.e.*,  $\hat{x}' \leq \hat{x}$ ) if the inner integral is positive and it must be weakly right to left (*i.e.*,  $\hat{x}' \geq \hat{x}$ ) if the inner integral is negative. This establishes the first part of the lemma.

To establish the second part, because  $f(\cdot, y)$  is a differentiable function for all  $y$ , if  $\tilde{x}$  is an interior maximum, then it must satisfy the first-order condition

$$0 = f_1(\tilde{x}, y).$$

Because  $f_1(\tilde{x}, \cdot)$  is strictly monotone,  $f_1(\tilde{x}, y) \neq f_1(\tilde{x}, y')$ ,  $y \neq y'$ . Hence,  $\tilde{x}$  does not satisfy the necessary first-order condition to maximize  $f(\cdot, y')$ . Therefore,  $\hat{x}' \neq \hat{x}$ ; that is, the inequalities are strict. ■

**Proof of Lemma 1:** The expression  $-c(q_1) - c(q_2)$  is Schur concave in  $q_1$  and  $q_2$  given that  $c(\cdot)$  is convex (see, *e.g.*, Marshall and Olkin, 1979, p. 64). The set of Schur concave functions is closed under addition and positive scalar multiplication, so

$$\zeta \beta(q_1, q_2) - c(q_1) - c(q_2)$$

is Schur concave in  $q_1$  and  $q_2$ . Result (i) then follows from the definition of Schur concavity.

Given (i), we know that if  $(q_1, q_2)$  maximizes (3), then  $q_1 = q_2$ . To prove (ii) observe that  $(0, 0)$  is not a solution to the program of maximizing (3) with respect to  $q_1$  and  $q_2$  given that its gradient evaluated at  $(0, 0)$  is

$$\zeta \times \left( \frac{\partial \beta(0, 0)}{\partial q_1}, \frac{\partial \beta(0, 0)}{\partial q_2} \right) > (0, 0)$$

(recall  $c'(0) = 0$ ). Consider the program

$$\max_{\{q_1, q_2\}} 2\beta(q_1, q_2) - c(q_1) - c(q_2).$$

In light of part (i), this program is equivalent to

$$\max_q 2\beta(q, q) - 2c(q). \quad (13)$$

Evaluated at  $q = 0$ , that expression is zero. Expression (13) is smaller for any  $q > q_M$  than when evaluated at  $q_M$  by condition (1). Hence, there is no loss of generality in restricting attention to  $q \in [0, q_M]$ . Given that domain is compact and (13) is continuous and bounded on  $[0, q_M]$ , a maximizer must exist by a theorem of Weierstrass's. As shown that maximizer is finite, but not zero. This maximizer can be labeled  $q^*(2)$ . From Lemma A.1 for any  $q > q^*(2)$  we have

$$\zeta\beta(q, q) - 2c(q) < \zeta\beta(q^*(2), q^*(2)) - 2c(q^*(2)). \quad (14)$$

Hence, there is no loss of generality when maximizing the left-hand side of (14) to limiting  $q$  to the interval  $[0, q^*(2)]$ . Invoking Weierstrass's theorem again, a maximum must exist on that interval. This is  $q^*(\zeta)$ .

Result (iii) and (v) follow immediately from Lemma A.1.

Consider (iv). Without loss of generality, hold  $q_2$  fixed. Suppose (iv) were false, then there would exist a  $\zeta$  and  $q_2$  such that the set

$$\left\{ q_1 \mid \zeta \frac{\partial\beta(q_1, q_2)}{\partial q_1} - c'(q_1) > 0 \right\}$$

is unbounded above. So for any  $\bar{q}_1$ , there exists a  $q_1 > \bar{q}_1$  such that

$$\zeta \frac{\partial\beta(q_1, q_2)}{\partial q_1} - c'(q_1) > 0. \quad (15)$$

Consider a  $q_1 > \max\{q_M, q_2\}$  that satisfies (15). A function  $\beta$  is Schur concave if and only if

$$(x_1 - x_2) \left( \frac{\partial\beta(x_1, x_2)}{\partial x_1} - \frac{\partial\beta(x_1, x_2)}{\partial x_2} \right) \leq 0$$

(see Marshall and Olkin, 1979, Theorem A.4, p. 57). Hence, since  $c(\cdot)$  is strictly convex, it follows from (15) that, for all  $q < q_1$ ,

$$\zeta \frac{\partial\beta(q_1, q)}{\partial q_2} - c'(q) > 0.$$

By continuity, letting  $q \rightarrow q_1$ , we have

$$\zeta \frac{\partial\beta(q_1, q_1)}{\partial q_2} - c'(q_1) \geq 0.$$

Given symmetry, this, in turn, implies

$$0 \leq 2 \frac{\partial\beta(q_1, q_1)}{\partial q_1} - c'(q_1). \quad (16)$$

Given that  $q_1 > q_M$ , expression (16) contradicts condition (1). The result follows *reductio ad absurdum*. ■

**Proof of Lemma 2:** By definition of an optimum:

$$\zeta\beta\left(q^{\text{BR}}(q^*(\zeta), \zeta), q^*(\zeta)\right) - c\left(q^{\text{BR}}(q^*(\zeta), \zeta)\right) \geq \zeta\beta(q, q^*(\zeta)) - c(q)$$

for all  $q$ ; hence,

$$\zeta\beta\left(q^{\text{BR}}(q^*(\zeta), \zeta), q^*(\zeta)\right) - c\left(q^{\text{BR}}(q^*(\zeta), \zeta)\right) \geq \zeta\beta(q^*(\zeta), q^*(\zeta)) - c(q^*(\zeta)).$$

Subtracting  $c(q^*(\zeta))$  from both sides yields

$$\begin{aligned} \zeta\beta\left(q^{\text{BR}}(q^*(\zeta), \zeta), q^*(\zeta)\right) - c\left(q^{\text{BR}}(q^*(\zeta), \zeta)\right) - c(q^*(\zeta)) \\ \geq \zeta\beta(q^*(\zeta), q^*(\zeta)) - c(q^*(\zeta)) - c(q^*(\zeta)). \end{aligned} \quad (17)$$

But, by definition,  $q^*(\zeta)$  maximizes (3) and moreover, by Schur concavity, no pair  $(q, q^*(\zeta))$ ,  $q \neq q^*(\zeta)$  can; hence, (17) can hold only if  $q^{\text{BR}}(q^*(\zeta), \zeta) = q^*(\zeta)$ . ■

**Proof of Proposition 2:** Lemmas 2 and 3 imply that both parties will honor the contract  $\langle q^*(\theta + 1), q^*(\theta + 1) \rangle$  in equilibrium.

To show this is the only such contract, we first establish that no contract  $\langle \bar{q}_1, \bar{q}_2 \rangle$  will be honored in equilibrium if  $\bar{q}_1 \neq \bar{q}_2$ . Without loss of generality, take  $\bar{q}_2 > \bar{q}_1$ . The function  $\beta$  is Schur concave if and only if

$$(\bar{q}_2 - \bar{q}_1) \left( \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_2} - \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_1} \right) \leq 0$$

(see Marshall and Olkin, 1979, Theorem A.4, p. 57). Hence,

$$\frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_2} \leq \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_1}. \quad (18)$$

Suppose it were the case that  $\langle \bar{q}_1, \bar{q}_2 \rangle$  would be honored in equilibrium, then they must be best responses and, so, solve the first-order conditions

$$(\theta + 1) \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_1} - c'(\bar{q}_1) = 0 \quad \text{and} \quad (19)$$

$$(\theta + 1) \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_2} - c'(\bar{q}_2) = 0. \quad (20)$$

Combining expressions (18)–(20) yields

$$c'(\bar{q}_1) = (\theta + 1) \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_1} \geq (\theta + 1) \frac{\partial\beta(\bar{q}_1, \bar{q}_2)}{\partial q_2} = c'(\bar{q}_2) > c'(\bar{q}_1),$$

where the last inequality follows because  $c(\cdot)$  is strictly convex. But this implies the contradiction  $c'(\bar{q}_1) > c'(\bar{q}_2)$ . *Reductio ad absurdum*, no contract will be honored in equilibrium if  $\bar{q}_1 \neq \bar{q}_2$ .

Observe, we have so far not required Assumption 1.

Finally, we wish to rule out the parties honoring a contract  $\langle \bar{q}, \bar{q} \rangle$ ,  $\bar{q} \neq q^*(\theta + 1)$ . Suppose, to the contrary, that there were such a contract. Then, from (19) and (20), it would be a local maximum of

$$(\theta + 1)\beta(q, q) - 2c(q).$$

But because that univariate function is strictly concave by Assumption 1, the function has only one maximum, which implies  $\bar{q} = q^*(\theta + 1)$ . ■

**Proof of Proposition 3:** Let

$$\pi^* = \beta(q^*(2), q^*(2)) - c(q^*(2)) \quad \text{and} \quad \pi_{\text{DEV}} = \max_q \beta(q, q^*(2)) - c(q).$$

By Proposition 1 (revealed preference, really),  $\pi_{\text{DEV}} > \pi^*$ . By Proposition 2,  $\pi^* > \pi_{\text{CON}}(\theta)$ . By continuity, there exists a  $\delta$  such that

$$\pi^* = \delta\pi_{\text{CON}}(\theta) + (1 - \delta)\pi_{\text{DEV}}. \quad (21)$$

Because  $\pi_{\text{DEV}} > \pi_{\text{CON}}(\theta)$  the right-hand side of (21) is decreasing in  $\delta$ , so the solution is unique. Call the solution  $\delta^*(\theta)$ . Because  $\pi_{\text{CON}}(\theta) < \pi^*$ ,  $\delta^*(\theta) < 1$ . If  $\delta \geq \delta^*(\theta)$ , then the left-hand side of (21) is greater, so (9) holds. Similarly, if  $\delta < \delta^*(\theta)$ , then (9) fails. Because, given Assumption 1,  $\pi_{\text{CON}}(\cdot)$  is an increasing function, it follows that, to maintain equality in (21),  $\delta$  must increase; that is,  $\delta^*(\cdot)$  is increasing. The limit result follows because  $\pi_{\text{CON}}(1) = \pi^*$ . ■

**Proof of Lemma 4:** Understanding the proof is facilitated by considering Figure 4. Rewrite (9) as

$$\beta(\hat{q}, \hat{q}) - c(\hat{q}) - (1 - \delta) \left( \max_q \beta(q, \hat{q}) - c(q) \right) \geq \delta\pi_{\text{CON}}(\theta). \quad (9')$$

Let  $\hat{q}(\delta)$  maximize the left-hand side of (9') (to establish a maximum exists, one can apply the argument used in the proof of Lemma 1(iv)). By the implicit function theorem,  $\hat{q}(\cdot)$  is continuous. By Lemma A.1,  $\hat{q}(\cdot)$  is increasing in  $\delta$ .

Observe  $\hat{q}(1) = q^*(2)$ , from which it follows that the left-hand side of (9') is strictly greater than the right-hand side if  $\delta = 1$ . Suppose  $\delta = 0$ , revealed preference implies the left-hand side of (9') is non-positive. It can be made to equal zero if  $\hat{q} = q^*(1)$ . Hence, the left-hand side and right-hand sides of (9') are equal at  $\delta = 0$ .

The derivative of the right-hand side of (9') with respect to  $\delta$  is  $\pi_{\text{CON}}(\theta)$ . Except if  $\theta = 0$ , this exceeds the derivative of left-hand side with respect to  $\delta$  evaluated at  $\delta = 0$ , which is

$$\beta(q^*(1), q^*(1)) - c(q^*(1)) = \pi_{\text{CON}}(0).$$

Hence, increasing  $\delta$  from 0 to some arbitrarily small  $\varepsilon > 0$  means causing the right-hand side of (9') to exceed the left. As noted, however, for  $\delta$  large enough the left-hand side exceeds the right-hand side. In other words, we have established that, for  $\theta > 0$ , the left-hand side of (9') as a function of  $\delta$  must cross the right-hand side as a function of  $\delta$  at least once at some  $\delta \in (0, 1)$ .

We wish to show that there is only one such crossing. Given the analysis above, we know, at the first such crossing, that the left-hand side crosses the right-hand side from below; hence, the slope of the left-hand side must exceed that of the right-hand side. There can, thus, be only one such crossing if we can show that the slope of the left-hand side remains greater than the slope of the right-hand side for all to the right of their point of crossing. Using the envelope theorem, the derivative of the slope of the left-hand side is

$$\frac{\partial}{\partial \hat{q}} \left( \max_q \beta(q, \hat{q}) - c(q) \right) \hat{q}'(\delta) = \frac{\partial \beta(q, \hat{q})}{\partial \hat{q}} \hat{q}'(\delta) \geq 0.$$

In other words, the slope of the left-hand side is non-decreasing in  $\delta$ . Since the slope of the right-hand side is a constant,  $\pi_{\text{CON}}(\theta)$ , it follows that the two sides cross once and only once at a  $\delta > 0$  provided  $\theta > 0$ . Define  $\hat{\delta}(\theta)$  as the value of  $\delta > 0$  at which they cross. See Figure 4.

For  $\delta < \hat{\delta}(\theta)$ , the right-hand side of (9') is greater than the maximum possible value of the left-hand side, hence no  $q > q^*(\theta + 1)$  can be sustained via a relational contract. For  $\delta \geq \hat{\delta}(\theta)$ ,  $\hat{q}(\delta)$ , at least, can be sustained via a relational contract. If  $\hat{q}(\delta) \leq q^*(\theta + 1)$ , then the left-hand side would necessarily be less than the right-hand side. Hence, for  $\delta > \hat{\delta}(\theta)$ ,  $\hat{q}(\delta) > q^*(\theta + 1)$ .

Finally, to see that  $\hat{\delta}(\cdot)$  is increasing consider Figure 4. If  $\theta$  increases, the line labeled RHS of (9') rotates counter-clockwise about the origin. It hence intersects the curve labeled LHS of (9') at a point to the right of the one illustrated. That  $\lim_{\theta \rightarrow 1} \hat{\delta}(\theta) = 1$  can also be seen from the figure given that  $\pi_{\text{CON}}(1) = \beta(q^*(2), q^*(2)) - c(q^*(2))$  and the curves cross once for  $\delta > 0$ . ■

**Proof of Lemma 5:** The proof requires five steps.

**Step 1:** We first establish that any solution to

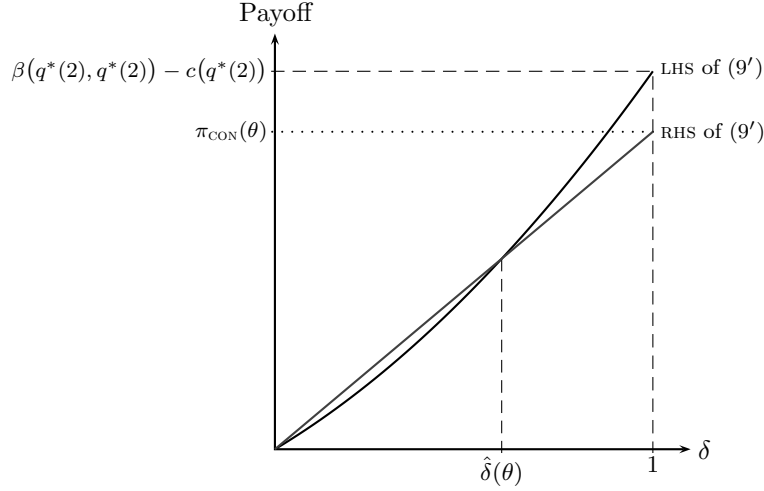
$$\max_q (\theta + 1)\beta(q, \hat{q}) - c(q) \tag{22}$$

is less than  $\hat{q}$ . By the assumptions of the lemma and the implicit function theorem, there exists a  $\hat{\theta} > \theta$  such that  $\hat{q} = q^*(\hat{\theta} + 1)$ . By Lemmas 2 and 3,  $\hat{q}$  therefore maximizes

$$\max_q (\hat{\theta} + 1)\beta(q, \hat{q}) - c(q). \tag{23}$$

By Lemma A.1, the solution to (22) is less than the solution to (23). Let  $\tilde{q}$  denote the solution to (22).

**Step 2:** Because  $\partial \beta(q, \hat{q}) / \partial q > 0$  for  $q \leq \hat{q}$ ,  $\beta(\tilde{q}, \hat{q}) < \beta(\hat{q}, \hat{q})$ .



**Figure 4:** Illustration for the Proof of Lemma 4. Figure valid for  $\theta \in (0, 1)$ . LHS and RHS denote left and right-hand sides, respectively.

**Step 3:** Using the envelope theorem,

$$\begin{aligned} \frac{\partial}{\partial \theta} \left( \max_q (\theta + 1)\beta(q, \hat{q}) - \theta\beta(\hat{q}, \hat{q}) - c(q) \right) \\ = \beta(\tilde{q}, \hat{q}) - \beta(\hat{q}, \hat{q}) < 0. \end{aligned} \quad (24)$$

**Step 4:** It follows, therefore, that

$$\begin{aligned} (1 - \delta) \left( \max_q (\theta + 1)\beta(q, \hat{q}) - \theta\beta(\hat{q}, \hat{q}) - c(q) \right) \\ < (1 - \delta) \left( \max_q \beta(q, \hat{q}) - c(q) \right), \end{aligned} \quad (25)$$

given that the right-hand side of (25) is the left-hand side evaluated at  $\theta = 0$ .

**Step 5:** Given (9) is assumed to hold and the right-hand side of (9) is greater than the right-hand side of (10) by (25), it follows that (10) must hold as well. ■

**Proof of Lemma 6:** The proof is effectively the same as the proof of Proposition 3. Let  $\pi^*$  have the same value as there. Now define

$$\pi_{\text{DEV}}(\theta) = \max_q (\theta + 1)\beta(q, q^*(2)) - \theta\beta(q^*(2), q^*(2)) - c(q).$$

For the same reasons given in the proof of Proposition 3,  $\pi_{\text{DEV}}(\theta) > \pi^* > \pi_{\text{CON}}(\theta)$ . Hence, a  $\delta$  exists that solves (21) given this new definition of  $\pi_{\text{DEV}}(\theta)$ . The existence of the cutoff value  $\delta^{**}(\theta)$  follows the logic given in that earlier proof.

We turn now to proving the “furthermore” sentence. Observe  $\pi_{\text{DEV}}(\theta)$  is the same as in the proof of Proposition 3 if  $\theta = 0$ . Hence,  $\delta^{**}(0) = \delta^*(0)$ . Finally, from Step 3 of the proof of Lemma 5,  $\pi_{\text{DEV}}(\theta)$  is here decreasing in  $\theta$ ; hence, the value of  $\pi_{\text{DEV}}(\theta)$  here is less than corresponding value in the proof of Proposition 3, from which it follows that the  $\delta$  that solves (21) is less here than in Proposition 3 for  $\theta \in (0, 1)$ .  $\blacksquare$

**Proof of Proposition 4:** The proof is by construction. Let  $\pi_{\text{DEV}}(\theta)$  be the same as in the proof of Lemma 6. Observe  $\delta^{**}(\theta)$  is the solution to

$$\beta(q^*(2), q^*(2)) - c(q^*(2)) = \delta\pi_{\text{CON}}(\theta) + (1 - \delta)\pi_{\text{DEV}}(\theta).$$

Hence,

$$\delta^{**}(\theta) = \frac{\beta(q^*(2), q^*(2)) - c(q^*(2)) - \pi_{\text{DEV}}(\theta)}{\pi_{\text{CON}}(\theta) - \pi_{\text{DEV}}(\theta)}.$$

It follows that

$$\begin{aligned} \text{sign}(\delta^{**'}(\theta)) = \text{sign} & \left( \left( \beta(q^*(2), q^*(2)) - c(q^*(2)) \right) (\pi'_{\text{DEV}}(\theta) - \pi'_{\text{CON}}(\theta)) \right. \\ & \left. - \pi_{\text{CON}}(\theta)\pi'_{\text{DEV}}(\theta) + \pi'_{\text{CON}}(\theta)\pi_{\text{DEV}}(\theta) \right). \quad (26) \end{aligned}$$

For case (i), (26) is positive.<sup>14</sup> For case (ii), calculations (available upon request) reveal that  $\delta^{**}(\theta) \equiv 1/2$ , which is constant. Finally, for case (iii), calculations (available upon request) reveal that

$$\delta^{**'}(\theta) = \frac{-1}{(2 + \theta)^2} < 0.$$

$\blacksquare$

**Proof of Proposition 5:** Define

$$\pi_{\text{DEV}}(\theta, \hat{q}) \equiv \max_q (\theta + 1)\beta(q, \hat{q}) - \theta\beta(\hat{q}, \hat{q}) - c(q).$$

From Lemma 2,

$$\begin{aligned} \pi_{\text{DEV}}(\theta, q^*(\theta + 1)) &= \beta(q^*(\theta + 1), q^*(\theta + 1)) - c(q^*(\theta + 1)) \\ &= \pi_{\text{CON}}(\theta). \end{aligned}$$

<sup>14</sup>The Mathematica program that verifies this is available from the authors upon request.

Hence, (10) is an equality for all  $\theta$  and  $\delta$  if  $\hat{q} = q^*(\theta)$ . The result follows, therefore, if we can show the left-hand side of

$$\beta(\hat{q}, \hat{q}) - c(\hat{q}) = \delta\pi_{\text{CON}}(\theta) + (1 - \delta)\pi_{\text{DEV}}(\theta, \hat{q}) \quad (27)$$

is increasing in  $\hat{q}$  faster than the right-hand side of that expression starting from  $\hat{q} = q^*(\theta + 1)$ . The derivative of the left-hand side of (27) evaluated at  $\hat{q} = q^*(\theta + 1)$  is

$$\begin{aligned} \frac{\partial\beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q_1} + \frac{\partial\beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q_2} - c'(q^*(\theta + 1)) \\ = 2\frac{\partial\beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q} - c'(q^*(\theta + 1)) \end{aligned} \quad (28)$$

$$= (1 - \theta)\frac{\partial\beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q}, \quad (29)$$

where (28) follows from symmetry and (29) follows from Proposition 2 (*i.e.*, that  $q^*(\theta + 1)$  satisfies the first-order condition for a party's optimal response to the other party choosing  $q^*(\theta + 1)$  under a formal contract). The derivative of the right-hand side of (27) reduces, using the envelope theorem and evaluated at  $\hat{q} = q^*(\theta + 1)$ , to

$$(1 - \theta)(1 - \delta)\frac{\partial\beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q}. \quad (30)$$

Because  $\delta > 0$ , (29) exceeds (30), as was to be shown.  $\blacksquare$

**Table A.1:** Summary Table of Notation

$\beta(q_1, q_2)$ : a symmetric function, is the gross benefit a party receives as a function of the two parties' level of cooperation.

$c(q)$ : is the cost of cooperation.

$D$ : damages paid the prevailing side if the parties go to court

$\delta$ : discount factor;  $\delta^*(\theta)$  is the cutoff value for the first-best outcome to be possible under relational contracting in either-or case;  $\delta^{**}(\theta)$  is the cutoff value for the first-best outcome to be possible under relational contracting when simultaneous use of both contract types are feasible;  $\hat{\delta}(\theta)$  is the cutoff level for a level of cooperation greater than that supportable by formal contracting alone to be feasible.

$\pi_{\text{CON}}(\theta)$ : one-period payoff using formal contracts only when quality of legal system is  $\theta$

$\Pi$ : party payoff

$q$ : amount of cooperation;  $q^*(\zeta)$  maximizes expression (3);  $q^*(2)$  is first-best level of cooperation.  $q^{\text{BR}}(q, \zeta)$  is a party's best-response to the other party choosing  $q$  given effective weight  $\zeta$ .

$\theta$ :  $\theta \in [0, 1)$  is the quality of the legal system.

$\zeta$ :  $\zeta \in (0, 2]$  is the effective weight placed on a party's benefit,  $\beta$ .

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